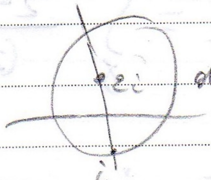
2618

Λ8


ακτίνα 3

$$f(z)=\frac{z}{(z+i)^2}$$ για $|z-2i|<3$

$$\frac{1}{z+i}=\frac{1}{z-2i+3i}=\frac{1/3i}{\frac{z-2i}{3i}+1}=\frac{1}{3i}\,\frac{1}{1+\frac{z-2i}{3i}}=$$

$$=\frac{1}{3i}\sum_{\nu=0}^{+\infty}(-1)^\nu\frac{(z-2i)^\nu}{(3i)^\nu}=\sum\frac{(-1)^\nu}{(3i)^{\nu+1}}(z-2i)^\nu \Rightarrow$$

παραγωγίζω

$$\frac{1}{(z+i)^2}=-\sum_{\nu=1}^{\infty}\nu\frac{(-1)^\nu}{(3i)^{\nu+1}}(z-2i)^{\nu-1}=\sum_{\mu=0}^{\infty}(\mu+1)\frac{(-1)^\mu}{(3i)^{\mu+2}}(z-2i)^\mu$$

$$\frac{z}{(z+i)^2}=\frac{z-2i+2i}{(z+i)^2}=\frac{z-2i}{(z+i)^2}+\frac{2i}{(z+i)^2}=\sum_{\mu=0}^{\infty}(\mu+1)\frac{(-1)^\mu}{(3i)^{\mu+2}}(z-2i)^{\mu+1}$$

$$+2i\sum_{\mu=0}^{\infty}(\mu+1)\frac{(-1)^\mu}{(3i)^{\mu+2}}(z-2i)^\mu$$

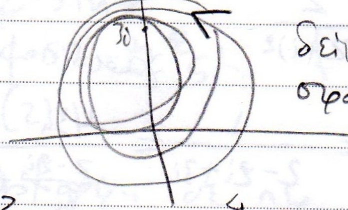
για $|z-2i|>3$:

$$\frac{3i}{z-2i}<1$$

$$\frac{1}{z+i}=\frac{\frac{1}{z-2i}}{\frac{3i}{z-2i}}=\frac{1}{z-2i}\sum_{\nu=0}^{\infty}(-1)^\nu\frac{(3i)^\nu}{(z-2i)^\nu}=\sum_{\nu=0}^{\infty}(-1)^\nu\frac{(3i)^\nu}{(z-2i)^{\nu+1}}$$

Ομοίως βρίσκω και το ανάπτυγμα

$$I = \int_\gamma \frac{z^2+\sin z}{(z-3i)^{50}}\,dz$$ κατά μήκος της καμπύλης γ



δεξιόστροφη στροφή στο $3i$

είναι 4

4 στροφές γύρω από το $3i$

Γενικά:

$$I(\gamma_1, z_0) f^{(\nu)}(z_0) = \frac{\nu!}{2\pi i}\int_\gamma \frac{f(z)}{(z-z_0)^{\nu+1}}\,dz$$

$$\Rightarrow 4 f^{(49)}(z_0) = \frac{49!}{2\pi i}\int_\gamma \frac{z^2+\eta\mu z}{(z-3i)^{50}}\,dz =$$

$$\left[\frac{4\cdot 2\pi i}{49!}\left(z^2+\sin z\right)^{(49)}\right]_{z=3i} = \frac{8\pi i}{49!}\sin\left(3i+49\frac{\pi}{2}\right)$$

$$I = \int_0^{2\pi}\frac{dt}{\sin x+\cos x+2} \xrightarrow{z=e^{it},\ t\in[0,2\pi]} \int_\gamma \frac{\frac{1}{iz}\,dz}{\frac{z-\frac{1}{z}}{2i}+\frac{z+\frac{1}{z}}{2}+2}$$